Find Ksp Given Solubility - Question 1

Find the Ksp for calcium carbonate given that the **solubility** of calcium carbonate is found to be 6.934 p.p.m.
(note p.p.m. = mg/L)

Must convert p.p.m. to mol/L first

$$\frac{6.934 \text{ mg } CaCO_3}{1 \text{ L}} \text{ x } \frac{1 \text{ g}}{1000 \text{ mg}} \text{ x } \frac{1 \text{ mol } CaCO_3}{100.09 \text{ g } CaCO_3} = \frac{6.93 \text{ x } 10^{-5} \text{ mol } CaCO_3}{1 \text{ L}}$$

$$CaCO_3(s) \rightleftharpoons Ca^{2+} + {CO_3}^{2-}$$

$$[Ca^{2+}] = 6.93 \text{ x } 10^{-5} \text{ M}$$
$$[{CO_3}^{2-}] = 6.93 \text{ x } 10^{-5} \text{ M}$$

$$\begin{aligned} K_{sp} &= [Ca^{2+}][{CO_3}^{2-}] \\ &= (6.93 \text{ x } 10^{-5})^2 \\ &= 4.8 \text{ x } 10^{-9} \end{aligned}$$

Please note that solubility must be in mol/L in order to find ion concentrations that can be used to solve for a Ksp value.

Find Solubility Ksp Given - Question 2

Find equilibrium ion concentrations for a saturated aluminum hydroxided solution and the mass of aluminum hydroxide in mg required to saturate 2500 mL of solution.

$$Al(OH)_3(s) \rightleftharpoons Al^{3+} + 3\,OH^{1-} \qquad Ksp = 3 \text{ x } 10^{-33}$$

Let s represent the solubility of $Al(OH)_3$

$$[Al^{3+}] = s$$
$$[OH^{1-}] = 3s$$

$$K_{sp} = [Al^{3+}][OH^{1-}]^3$$
$$3 \text{ x } 10^{-33} = (s)(3s)^3$$
$$3 \text{ x } 10^{-33} = (s)(27s^3)$$
$$3 \text{ x } 10^{-33} = 27s^4$$
$$s = 3.25 \text{ x } 10^{-9} \text{ mol/L}$$

$$[Al^{3+}] = 3.25 \text{ x } 10^{-9} \text{ M}$$

$$[OH^{1-}] = 3 \text{ x } (3.25 \text{ x } 10^{-9} \text{ M})$$
$$= 9.74 \text{ x } 10^{-9} \text{ M}$$

$$2500 \text{ mL x } \frac{1 \text{ L}}{1000 \text{ mL}} \text{ x } \frac{3.25 \text{ x } 10^{-9} \text{ mol}}{1 \text{ L}} \text{ x } \frac{78.01 \text{ g } Al(OH)_3}{1 \text{ mol } Al(OH)_3} \text{ x } \frac{1000 \text{ mg}}{1 \text{ g}} = 6.34 \text{ mg } Al(OH)_3$$

Find Maximum Ion Concentration Given a Fixed Ion Concentration – Question 3

Find the maximum concentration of Pb^{2+} in p.p.m. that can co-exist in equilibrium with a chloride ion concentration of 0.05 M.

$$Pb(Cl)_2 \rightleftharpoons Pb^{2+} + 2\,Cl^{1-} \qquad Ksp = 1.6 \text{ x } 10^{-5}$$

$$K_{sp} = [Pb^{2+}][Cl^{1-}]^2$$

$$[Pb^{2+}] = \frac{K_{sp}}{[Cl^{1-}]^2}$$

$$[Pb^{2+}] = \frac{1.6 \text{ x } 10^{-5}}{(0.05)^2}$$

$$[Pb^{2+}] = 6.4 \text{ x } 10^{-3} \text{ M}$$

$$\frac{6.4 \text{ x } 10^{-3} \text{ mol } Pb^{2+} \text{ L}}{1 \text{ L}} \text{ x } \frac{207.20 \text{ g } Pb^{2+}}{1 \text{ mol } Pb^{2+}} \text{ x } \frac{1000 \text{ mg}}{1 \text{ g}} = \frac{1326 \text{ mg } Pb^{2+}}{1 \text{ L}}$$

$$\therefore [Pb^{2+}] = 1326 \text{ p.p.m.}$$

Find Ion Concentration in a Solution of a Soluble Salt (another fixed ion problem) - Question 4

what is the maximum possible concentration of magnesium ion in p.p.m. that can exist in a 1.0 M $K_2C_2O_4$ solution

$$\underset{1.0\ M}{K_2C_2O_4} \longrightarrow 2\,K^{1+} + \underset{1.0\ M}{C_2O_4{}^{2-}}$$

$$MgC_2O_4 \rightleftharpoons Mg^{2+} + \underset{1.0\ M}{C_2O_4{}^{2-}} \qquad Ksp = 8.6 \text{ x } 10^{-5}$$

$$K_{sp} = [Mg^{2+}][C_2O_4{}^{2-}]$$

$$[Mg^{2+}] = \frac{K_{sp}}{[C_2O_4{}^{2-}]^2}$$

$$[Mg^{2+}] = \frac{8.6 \text{ x } 10^{-5}}{1.0}$$

$$[Mg^{2+}] = 8.6 \text{ x } 10^{-5} \text{ M}$$

$$\frac{8.6 \text{ x } 10^{-5} \text{ mol } Mg^{2+} \text{ L}}{1 \text{ L}} \text{ x } \frac{24.31 \text{ g } Mg^{2+}}{1 \text{ mol } Mg^{2+}} \text{ x } \frac{1000 \text{ mg}}{1 \text{ g}} = \frac{2.090 \text{ mg } Mg^{2+}}{1 \text{ L}}$$

$$\therefore [Mg^{2+}] = 2.090 \text{ p.p.m.}$$

Ksp Problem: Common Ion Effect

Determine the solubility of lead iodide in a 0.5M KI solution

common ion

$$\underset{0.5M}{KI} \rightarrow K^+ + \boxed{\underset{0.5M}{I^-}}$$

$$PbI_2 \rightleftharpoons Pb^{2+} + \boxed{2I^-}$$

Let s represent the solubility

$\therefore [Pb^{2+}] = s$

$[I^-] = 2s + 0.5$

↙ contribution from KI

↖ contribution from PbI_2

$K_{sp} = [Pb^{2+}][I^-]^2$

$7.1 \times 10^{-9} = s(2s+0.5)^2$

* use methods to solve a cubic or !!

assume $2s \ll\ll 0.5$ $\therefore 2s + 0.5 \simeq 0.5$

$7.1 \times 10^{-9} \simeq s(0.5)^2$

$s = 2.84 \times 10^{-8}$ mol/L $\quad \because 2.84 \cdot 10^{-8} \ll\ll 0.5$ (less than 1%)

$\therefore$ assumption was valid

Note the solubility will be lower than in pure water

Complex Ksp problem - determine mass of ppte formed.

400 mL of 0.0001 M $Ba(NO_3)_2$ is mixed with 500 mL of 0.0003 M Na_2CO_3. What mass of ppte forms in mg?

$Ba(NO_3)_2(s) \rightarrow Ba^{2+} + 2NO_3^{1-}$

$Na_2CO_3(s) \rightarrow 2Na^{+} + CO_3^{2-}$

ppte → $BaCO_3(s) \rightleftharpoons Ba^{2+} + CO_3^{2-}$ *

* Na^{1+} + NO_3^{1-} are highly soluble and hence will not be involved in a ppte

	$BaCO_3(s)$	Ba^{2+}	CO_3^{2-}
Initial [ ]	N.A.		
Initial amount	0	$n = CV$ $n = 0.0001M \times 0.4L$ $n = 4 \times 10^{-5}$ mol	$n = CV$ $n = 0.0003M \times 0.5L$ $n = 1.5 \times 10^{-4}$ mol
Final amount	x	$4 \times 10^{-5} - x$	$1.5 \times 10^{-4} - x$
Final [ ]	N.A.	$\frac{4 \times 10^{-5} - x}{0.9}$	$\frac{1.5 \times 10^{-4} - x}{0.9}$

400 mL + 500 mL = 900 mL

Total Volume: 0.9 L

Let x represent the amount of $BaCO_3$ that forms

$K_{sp} = [Ba^{2+}][CO_3^{2-}]$

$4.9 \times 10^{-9} = \left(\frac{4 \times 10^{-5} - x}{0.9}\right)\left(\frac{1.5 \times 10^{-4} - x}{0.9}\right)$ (from table)

"quadraticus" ⟹

~~extraneous~~ ~~$x = 1.78 \times 10^{-4}$ mol (too big)~~

or

$x = 1.137 \times 10^{-5}$ mol

$1.137 \times 10^{-5}\ mol\ BaCO_3 \times \frac{197.34\ g\ BaCO_3}{1\ mol\ BaCO_3} \times \frac{1000\ mg}{1\ g} = 2.244\ mg\ BaCO_3$

Complex Ksp problem - find Ksp value

eg 400 mL of 0.0001M $Ba(NO_3)_2$ is mixed with 500 mL of 0.0003M Na_2CO_3. 2.244 mg of ppte forms. Find Ksp for the ppte produced

$$Ba(NO_3)_2 \rightarrow Ba^{2+} + 2NO_3^{1-}$$

$$Na_2CO_3 \rightarrow 2Na^{+} + CO_3^{2-}$$

$$BaCO_{3(s)} \rightleftharpoons Ba^{2+} + CO_3^{2-}$$

Initial [ ]	/	/	/
Initial Amount	0	$n = CV$ $n = 0.0001\,M \times 0.4\,L$ $n = 4 \times 10^{-5}$ mol	$n = CV$ $n = 0.0003\,M \times 0.5\,L$ $n = 1.5 \times 10^{-4}$ mol
Final Amount	1.137×10^{-5} mol	$4 \times 10^{-5} - 1.137 \times 10^{-5} = 2.863 \times 10^{-5}$ mol	$1.5 \times 10^{-4} - 1.137 \times 10^{-5} = 1.386 \times 10^{-4}$ mol
Final [ ]	N.A.	$3.181 \times 10^{-5}\,M$	$1.5540 \times 10^{-4}\,M$

400 mL + 500 mL = 900 mL

∴ Total Volume = 0.9 L

$$2.244\,mg\ BaCO_3 \times \frac{1\,g}{1000\,mg} \times \frac{1\,mol\ BaCO_3}{197.34\,g\ BaCO_3} = 1.137 \times 10^{-5}\ mol\ BaCO_3$$

$$K_{sp} = [Ba^{2+}][CO_3^{2-}]$$

$$K_{sp} = (3.181 \times 10^{-5})(1.540 \times 10^{-4})$$

$$K_{sp} = 4.9 \times 10^{-9}$$