Unit Analysis Example:

Suppose you wish to use a complex equation such as:

$$E_{e^-} = \frac{-e^4 m}{8\varepsilon_o^2 h^2} \text{ x} \left(\frac{1}{n^2}\right)$$

given the following information:

e = 1.6022 x 10^{-19} C (fundamental unit of charge)
m = 9.110 x 10^{-31} kg (resting mass of an electron)
p = 3.1415926536
ε_o = 8.854 x 10^{-12} $C^2N^{-1}m^{-2}$ (dielectric constant)
h = 6.626 x 10^{-34} Js (Planck's constant)
n = 1 (no unit because it is a whole number deBroglie wavelengths)

One approach is to deal with the numbers and the units separately. This means doing a separate unit analysis!! Read On.

$$E_{e^-} = \frac{-e^4 m}{8\varepsilon_o^2 h^2} \text{ x} \left(\frac{1}{n^2}\right)$$

$$E_{e^-} = \frac{-\left(1.6022 \text{ x } 10^{-19}\right)^4 \text{ x } 9.11 \text{ x } 10^{-31}}{8 \text{ x } \left(8.854 \text{ x } 10^{-12}\right)^2 \text{ x } \left(6.626 \text{ x } 10^{-34}\right)^2} \text{ x} \left(\frac{1}{n^2}\right)$$

$$E_{e^-} = -2.18029 \text{ x } 10^{-18} \text{ J x} \left(\frac{1}{n^2}\right)$$

Note that the unit of J has mysteriously appeared at the end of this calculation. This is where the separate unit analysis comes in which should follow the format outlined on the next page. During this unit analysis it is necessary to replace composite units with their base units for example:

$$J = \frac{kgm^2}{s^2} \qquad N = \frac{kgm}{s^2}$$

The unit analysis itself makes intensive use of the laws of exponents and standard algebraic practice. The unit analysis is to the left, a comment on what was done is to the right. The comment is not required in your unit analysis on this assignment or up coming quiz/test. There is more than one correct way to work through this analysis

units = $\dfrac{C^4 kg}{\left(C^2 N^{-1} m^{-2}\right)^2 (Js)^2}$	plug in the units
= $\dfrac{C^4 kg}{C^4 N^{-2} m^{-4} J^2 s^2}$	clear the brackets (laws of exponents)
= $\dfrac{kg}{N^{-2} m^{-4} J^2 s^2}$	cancel the C^4
= $\dfrac{kg N^2 m^4}{J^2 s^2}$	multiply by $\dfrac{N^2 m^4}{N^2 m^4}$ to convert the negative exponents to positive
= $\dfrac{kg m^4}{s^2} \times \left(\dfrac{N}{1}\right)^2 \times \left(\dfrac{1}{J}\right)^2$	unnecessary, but separating the composite unit before the next step might help
= $\dfrac{kg m^4}{s^2} \times \left(\dfrac{kg m}{s^2}\right)^2 \times \left(\dfrac{s^2}{kg m^2}\right)^2$	substitute for the composite units of N and J
= $\dfrac{kg m^4}{s^2} \times \dfrac{kg^2 m^2}{s^4} \times \dfrac{s^4}{kg^2 m^4}$	clear the brackets (laws of exponents)
= $\dfrac{kg m^2}{s^2}$	massive cancellations (often helps to cross fingers before this step)
= J	substitution for composite unit!! We got the unit we want